Lab 8: Taylor Series

This lab uses *Mathematica* to investigate the Taylor series expansions for given functions. We'll consider only Taylor series expanded about x=0, also called *Maclaurin series*.

Copy the notebook from the **/math212/notebooks** directory into your own **notebooks** subdirectory; launch the *WriteNow* text processor to set up and save your lab report in your **reports** subdirectory; work through each section of the notebook and write a summary of what you learned in your report; mail your report to **math212**.

Be sure to work all the exercises in the notebook and to describe your results in your report.

Here are some ideas to consider when writing your report:

Taylor Polynomials

1. How can *Mathematica* produce Taylor polynomials for a given function?
2. How can you use *Mathematica* to compare a function with its Taylor polynomials?
3. What does your work tell you about the convergence of Taylor polynomials?

Taylor's Theorem

1. How can *Mathematica* be used to investigate the accuracy of the approximation of a function by its Taylor polynomials?
2. What is the connection between the degree of a Taylor polynomial and the maximum error incurred by using it to approximate a function on a given interval?

Mathematica Commands

Here are some of the *Mathematica* commands that you might use in this lab:

`Series[ f[x], {x,0,8} ]`	Produces the Taylor polynomial of degree n centered at x=0.
`Normal[%]`	Removes the asymptotic term, returning the pure polynomial.
`D[f[x],{x,n+1}]`	The (n+1)st derivative of f.
`/.x->c`	Evaluated at x=c.
`R[f][n,x,c]`	The remainder term for f from Taylor's Theorem (page 540).
`R[n_,x_] := x^(n+1)/(n+1)!`	Defines the remainder term for f, assuming that its (n+1)st derivative is bounded by 1.
`Table[ {n,R[n,N[Pi]]}, {n,1,20} ]`	Produces a table of 20 pairs: n and the nth remainder term at x=Pi.
`Table[{f[x],P[x],f[x]-P[x]},{x,3.00,3.15,0.01}]`	Produces a table of 16 triples: f[x], P[x], and their difference, allowing x to range from 3.00 to 3.15 in increments of 0.01.
`N[ t, 10]  //  TableForm`	Produces the table t in tabular form, with all numbers given to 10 decimal-place accuracy.